Extreme Beam Broadening Using Phase Only Pattern Synthesis

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***Abstract*— This paper describes methods for broadening the beam of a phased array antenna using phase-only element weights. This type of broadening can be valuable for improving the search occupancy and/or reducing the search frame time for large phased arrays, when transmit beam broadening is combined with multiple simultaneous receive beams. Phase-only broadening is required for typical solid-state (active phased) arrays because control of the transmit amplitude at each element is not practical. A notable result is that there is a relationship between the amount of beam broadening and the efficiency (directivity) of the resulting beam. In particular it is shown that broadening less than ~2.5:1 results in reduced beam efficiency for both linear and circular arrays, while a broadening of greater than ~2.5:1 results in improved beam efficiency, and that, therefore, certain broadening configurations will be preferable to the radar designer (e.g. 4:1 in one plane rather than 2:1 in two planes, due to a large difference in efficiency). The algorithms used to achieve the broadening patterns presented here are based on a combination of homotopy from known optimality with a stochastic gradient descent approach using a carefully constructed one parameter family of penalty functions.**

***Index Terms*— pattern synthesis, wideband array processing, transmit spoiling, null control.**

I. INTRODUCTION

The trend toward multifunction phased array antennas often leads to a beamwidth that is much smaller than would be ideal for conducting a given search operation. In some situations the antenna beam may be so narrow that search frame times become wholly impractical [1]. In situations such as this, a combination of a broadened transmit beam and multiple simultaneous receive beams may be utilized to alleviate the problem.

This paper describes methods for optimally broadening the transmit beam, parametrically explores broadening geometries for linear and circular apertures, and presents point design examples for a realistic element layout. For broadening factors (BF) larger than about 2.5 more power can be directed into the broadened mainbeam region than would be expected from a smaller array that achieves the same mainbeam coverage and transmits with the same power, a result that has interesting implications for broadening strategies in search and track.

The basic challenge in the *transmit pattern synthesis problem* for array architectures is to *achieve a desired radiation pattern through the setting of element level amplitude and phase weights*. For transmit operation, the problem is often additionally constrained by a lack of amplitude control, such as in modern solid-state phased arrays wherein the transmit amplifiers are operated in saturation and amplitude adjustment is not practical. In these cases, phase-only synthesis must be employed in the transmit beam broadening. A special case of the approach considered here for phase only pattern synthesis can be found in [2], while approaches to pattern synthesis using both amplitude and phase control can be found in references [3] and [4]. Two approximate approaches to the null-only pattern synthesis problem can be found in [5], considering unconstrained complex element weights, and [6], where phase only weighting is addressed. This short list should not be viewed as being anything close to exhaustive, but touches on some relevant previous work to the approach considered in the present paper.

As in any array signal processing problem, the level of difficulty associated with solving the pattern synthesis problem depends on the number of array element variables, the element spacing and layout, the waveform bandwidth, and the degree to which the objective radiation pattern differs from the natural beam shape. Small numbers of elements with uniform spacing and narrow bandwidths generally make for tractable problems. More challenging cases include large antennas with possibly tens of thousands of elements, wide bandwidths, fixed amplitude constraints, or requirements to adapt to dynamic platform and environmental conditions.

II. APPROACH

A typical phase only pattern synthesis (POPS) problem will include broadening of the main beam along with simultaneous signal suppression in some sidelobe region. It is not assumed here that the array under consideration has periodic element layout, that the broadening region is elliptical, or that the signal suppression region is rectangular, as there are natural situations in which all of these conditions are undesirable. For example, the horizon maps into sine-space as an arc when the face of the array is tilted such that the horizon is off array broadside. Thus, a horizon notch is best described as a non-linear envelope around the horizon, not a rectangular keep-out region.

To develop a mathematical description of the POPS problem, define a region B in sine space, called the *broadening region*, within which the goal is to find element level phase settings that will cause the radiated antenna pattern to remain within 3dB of the maximum pattern value. Similarly, define the *nulling region* N in sine space where the goal is to suppress the signal below some objective value M_N. We introduce two mathematical formulations of the POPS problem below; the first formulation describes an intuitive approach to the problem, while the second reflects the implementation used for the results in this paper.

Let

$$E(\mathbf{u};\varphi,a,\mathbf{x}) = \left| \sum_{k=1}^{N_e} a_k\, g_k(\mathbf{u})\, e^{j\left(2\pi(\mathbf{x}_k-\bar{\mathbf{x}})^T\mathbf{u}/\lambda+\varphi_k\right)} \right|^2$$

denote the power density at a point **u** in sine space for given element phase weightings φ_k, amplitude weightings a_k, and element positions $\mathbf{x}_k$. The term $\bar{\mathbf{x}}$ denotes the mean of the element positions.

Problem 1: Take the amplitude weights to be fixed, and seek to select ϕ_k values to achieve the objective conditions

$$M_B \le \min \left\{ E(\mathbf{u};\varphi,a) : \mathbf{u} \in B \right\}, \text{ and}$$

$$M_N \ge \max \left\{ E(\mathbf{u};\varphi,a) : \mathbf{u} \in N \right\}$$

for fixed M_B and M_N.

Problem 2: Define $E(\mathbf{u};\varphi,a)$ and M_N as above, and require that the condition, $\mathbf{C}^+$, that

$$M_B(\varphi) \le \min \left\{ E(\mathbf{u};\varphi,a) : \mathbf{u} \in B \right\}$$

is satisfied for the phase weighting φ, where

$$M_B(\phi) = 1/2 \max \left\{ E(\mathbf{u};\varphi,a) : \mathbf{u} \in B \right\}.$$

Assuming an elliptical region B in Problem 1, a natural choice of M_B is the 3dB value of the beam formed by a uniformly weighted aperture whose spatial dimensions give rise to the region B. Because of the larger aperture size, faster roll off is possible in the spoiled beam, leading to the possibility of packing more power into the main beam when solving the POPS problem. We show in several examples that this effect does not occur for small BF, but that the possibility is realized in both 1-D and 2-D broadening for larger BF; this fact has interesting consequences for the radar system design as discussed below.

We use the *beam efficiency* as a metric to determine how well a particular spoiled pattern performs against a nominal pattern. The beam efficiency is defined as the ratio of the energy in the sine space region enclosed by the 3dB contour of the beam under consideration to that of a uniformly weighted aperture the same total power as the beam under consideration whose 3dB contour is B.

III. BROADENING METHOD

The broadening method used here is based on a homotopy from the known optimal beam with BF=1. Local optimality is maintained in an objective functional designed to generate the desired beam conditions, as described below. Because the broadening phase values are generated by perturbation of an optimal solution, the resulting pattern is robust to element level phase errors; in short, perturbation solutions are perturbation stable.

The objective functional used here is based on decomposing the desired broadening region B into two disjoint regions; in the first, B^+, the power satisfies condition $\mathbf{C}^+$, while in the region B^- the power violates this condition. The objective function in this case attempts to maximize the power in the region B^+, while simultaneously penalizing the failure to achieve the desired power density in the region B^-. The cost function used for this analysis takes the form

$$F(\varphi;B) = \int_{\mathbf{u}\in B^+} E(\mathbf{u};\varphi,a)\, d\mathbf{u}$$

$$+\,\alpha \int_{\mathbf{u}\in B^-} \left(E(\mathbf{u};\varphi,a) - M_B(\phi)\right) d\mathbf{u},$$

where α is a scaling term.

IV. PARAMETRIC ANALYSIS

Broadening factors (BF) from 1 (no broadening) to 15 were computed for a linear aperture of length 50 wavelengths, and a circular aperture of radius $26\sqrt{\pi}$ wavelengths. The phase weights were computed using a homotopy technique, with the BF varied from 1 to 15 in discreet steps. For this method, the weights of one BF were used as a starting point for the next higher BF. A stochastic gradient decent technique was then used to optimize the cost functional discussed above. Results are shown in Figure 1 and Figure 2, and are discussed below. This technique can be applied to more realistic apertures (irregular, 7000 elements), examples of which are shown in Figure 5; experience shows that the cases presented here serve as good rules-of-thumb in predicting performance for the more realistic apertures.

Antenna gains of BF 1-5 may be seen in Figure 1 (a) and (b). They show respectively the antenna gains and normalized antenna gains. The axes in Figure 1(a) were selected to better illustrate the sidelobe structure. Note that the relative sidelobes for the spoiled beams may be considerably higher than those of the unspoiled beam. It is possible to add constraints to better control sidelobe performance, but only at the detriment of mainbeam performance, a tradeoff that would be examined in more depth based on operational constraints or performance considerations.

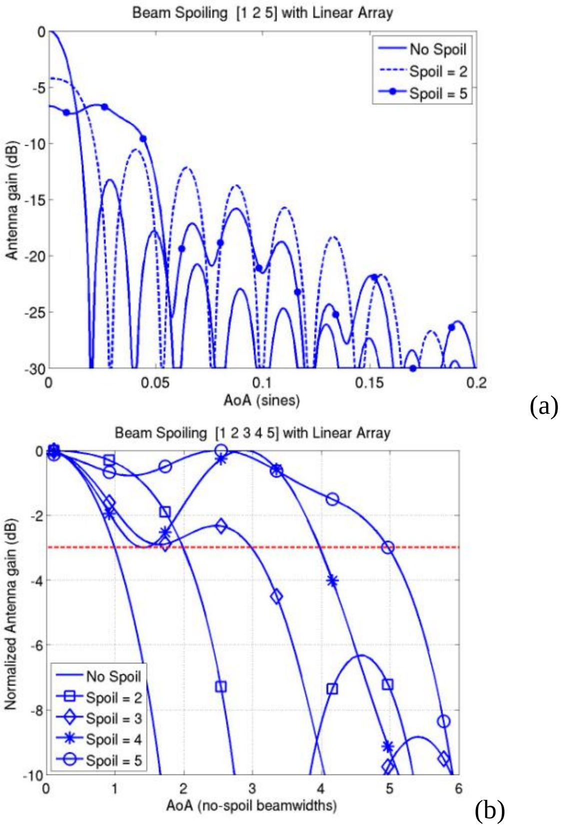


Figure 1 Pattern gains (a) and normalized gains (b) for integer broadening factors 1 to 5.

The main beam broadening may be better seen in Figure 1 (b), where the axes were selected to illustrate the mainbeam behavior with the x-axis in units of unspoiled beamwidths. One can clearly see that the beams are correctly spoiled as their 3-dB points intercept at the proper spoiled beamwidths. Figure 1 (b) illustrates a critical result of this analysis: that the beam efficiency achievable is directly related to the broadening factor.

As shown in the figure, broadening factors below approximately 2.5 result in reduced efficiency, whereas broadening factors greater than 2.5 result in *improved* efficiency. This improvement in efficiency is possible due to the additional degrees of freedom available in the larger aperture. There can be a very clear advantage in the geometry of beam broadening selected. For example, if a transmit beam of 4 times larger area is desired, the natural tendency would be to broaden the beam by a factor of 2 in each dimension, and as can be seen in Figure 2 (b) this will result in poor efficiency. If the beam is instead broadened by a factor of 4 in only one dimension, the total difference in efficiency can exceed 3 dB, a substantial benefit for the resultant search performance.

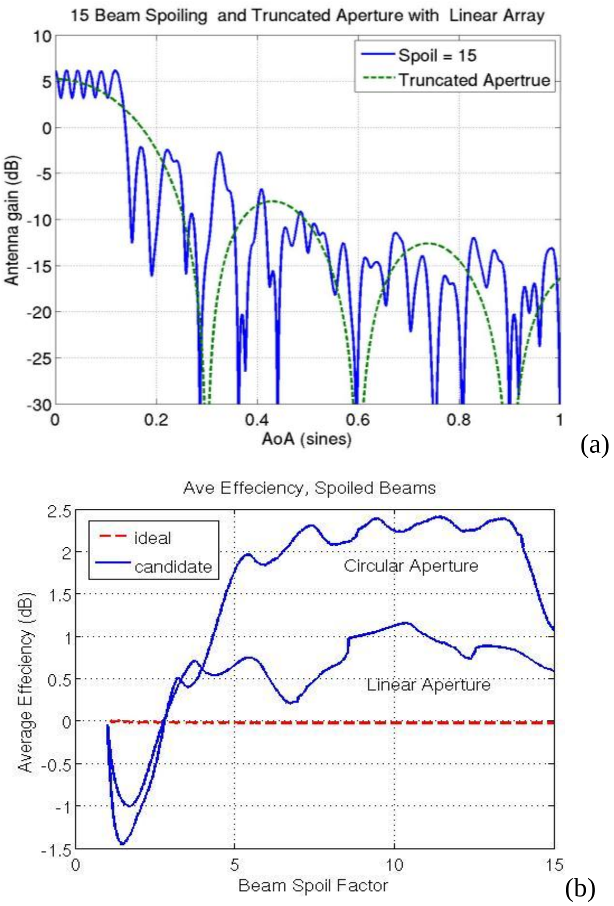


Figure 2 (a) Extreme broadening factor of 15 compared to the nominal aperture with the same beamwidth. (b) Efficiency vs. Broadening Factor from 1 to 15 beamwidths.

A comparison of an antenna pattern with a BF of 15 with an unspoiled aperture that has been truncated to produce an equivalent beamwidth can be seen in Figure 2. Note from Figure 2 (a) that the rapid roll-off of the broadened pattern allows for more energy in the mainbeam and a faster transition to the first null.

V. POINT DESIGN EXAMPLES

The techniques used above to examine efficiency of broadened patterns for linear and circular arrays can be directly applied to more general array layouts.

An element configuration typical of a realistic array is shown in Figure 3. The elements in this case have approximately ½ wavelength spacing and are arranged on a triangular lattice. The lengths in the x-axis dimension and y-axis dimension are not the same, and the array has rough edges that can be typical of manufacturing or mounting constraints.

The natural antenna pattern of this array, generated from a uniform element weighting, is shown in Figure 4 (a). An example of 4:1 spoiling for this same array using the phase-only technique described above is shown in Figure 4 (b). Both patterns are generated assuming the same energy per element, and distributing the same total energy over a larger area results in a reduction in the peak energy. The efficiency of this pattern, however, is 1.38 dB higher than would be expected from computing the main beam energy of an array radiating with the same total power, but only one quarter the aperture area. Notice that this performance is better than would be expected from the linear analysis of the previous section.

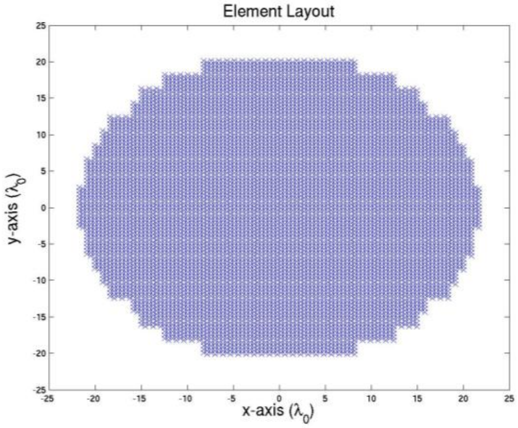
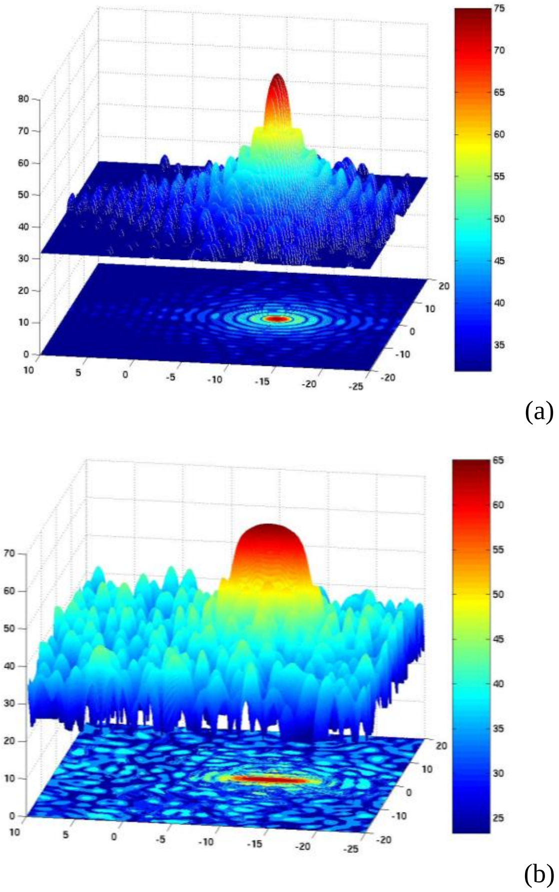


Figure 3 Array consisting of approximately 7000 elements arranged on a triangular lattice with irregular edges.



(a)

(b)

Figure 4 (a) Natural antenna pattern for the array shown in Figure 3 and (b) phase only synthesis broadened pattern with a 4:1 broadening factor.

Simultaneous broadening and nulling can be achieved by modifying the form of the one parameter family of cost functionals introduced above. In this case the homotopy gradually pushes energy out of the desired nulling region at the same time as the broadened beam is being formed. In Figure 5, we show an antenna pattern with 1.5:1.5 broadening and a 5 degree null trough achieving 40 dB null depth adjacent to the main beam. This case is shown as an example. Based on the analysis from the previous section, we suspect that a more efficient target distribution would likely be had with a 2.25:1 spoiling.

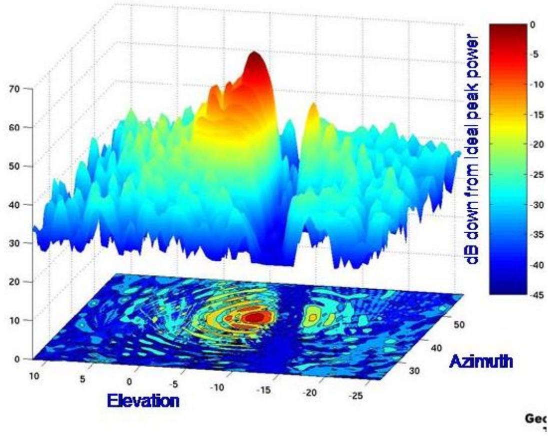


Figure 5 Broadening and wideband notch using the element layout in Figure 3. Pattern has 1.5:1.5 broadening with -40 dB null trough.

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